UKM 2021 VIII-3

pro $\frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)$

$$E = \frac{1}{2}\left(\frac{1}{2}\hbar\omega + \frac{5}{2}\hbar\omega\right) = \frac{3}{2}\hbar\omega \rightarrow \frac{\partial E}{\partial \omega} = \frac{3}{2}\hbar$$

$$\langle\psi| m\omega x^2 |\psi\rangle = \frac{1}{2}(\langle 0| + \langle 2|)\left[\frac{\hbar}{2}(a^2 + a^{+2} + 2a^+a + 1)\right](|0\rangle + |2\rangle)$$

$$= \frac{\hbar}{4}\Big[\langle 0| 2a^+a + 1 |0\rangle + \langle 2| 2a^+a + 1 |2\rangle$$ $\quad a^+a|2\rangle = a^+\sqrt{2}|1\rangle = 2|2\rangle$

$$+ \langle 0| a^2 |2\rangle + \langle 2| a^{+2} |0\rangle\Big]$$

$= \langle 0| a\sqrt{2}|1\rangle = \langle 0|\sqrt{2}|0\rangle = \sqrt{2}$ $\qquad = \langle 2| a^+\sqrt{2}|1\rangle = \langle 2|\sqrt{2}|2\rangle = \sqrt{2}$

$$= \frac{\hbar}{4}\left[1 + \cancel{3}\,5 + 2\sqrt{2}\right] = \frac{3}{2}\hbar + \frac{\hbar}{\sqrt{2}}$$ konečně správně!

Mějme LHO částici ve stavu $|\psi\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |3\rangle)$

Ověřte $\frac{d\langle A\rangle}{dt} = \frac{i}{\hbar}\langle [H, A]\rangle + \left\langle\frac{\partial A}{\partial t}\right\rangle$

pro $\hat{A} = \hat{p}$

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

$$[H, p] = \left[\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2, p\right]$$

$$= \left[\frac{p^2}{2m}, p\right] + \frac{1}{2}m\omega^2[x^2, p]$$

$$= 0 + \frac{1}{2}m\omega^2\left[x^2p - px^2\right]$$, známe $[x,p] = i\hbar$

$$x^2p - px^2 = xxp - pxx = \underbrace{xxp - xpx}_{x[x,p]} + \underbrace{xpx - pxx}_{[x,p]x}$$

$$= xi\hbar + xi\hbar = 2i\hbar x$$

$$[H, p] = \frac{1}{2}m\omega^2\, 2i\hbar x = im\omega^2\hbar x$$

$$= im\omega^2\hbar\,\frac{\alpha}{\sqrt{2}}(a + a^+)$$

$$p = \frac{\sqrt{\hbar m\omega}}{\sqrt{2}\,i}(a - a^+)$$

$$\langle p\rangle = \frac{1}{3}\left(\langle 0|e^{\frac{i\omega t}{2}} + \langle 1|e^{\frac{i3\omega t}{2}} + \langle 3|e^{\frac{i7\omega t}{2}}\right)\frac{\sqrt{\hbar m\omega}}{\sqrt{2}\,i}(a - a^+)\left(|0\rangle e^{-\frac{i\omega t}{2}} + |1\rangle e^{-\frac{3i\omega t}{2}} + |3\rangle e^{-\frac{7i\omega t}{2}}\right)$$

$$= \frac{1}{3}\frac{\sqrt{\hbar m\omega}}{\sqrt{2}\,i}\Big[\underbrace{\langle 0|a|1\rangle}_{1} e^{-i\omega t} - \underbrace{\langle 1|a^+|0\rangle}_{1} e^{i\omega t}\Big]$$

$$= \frac{1}{3}\sqrt{\hbar m\omega}\sqrt{2}\,\frac{1}{2i}(-1)\left[e^{i\omega t} - e^{-i\omega t}\right] = -\frac{\sqrt{2}}{3}\sqrt{\hbar m\omega}\sin(\omega t)$$

$$\frac{\partial\langle A\rangle}{\partial t} = -\frac{\sqrt{2}}{3}\sqrt{\hbar m\omega}\,\omega\cos(\omega t)$$

UKM 2021
VIII - 4

$$\frac{d\langle A\rangle}{dt} = \frac{i}{\hbar}\langle [H, A]\rangle + \underbrace{\left\langle \frac{\partial A}{\partial t}\right\rangle}_{\to 0}$$

$$= \frac{i}{\hbar}\left\langle i m\omega^2 \hbar \frac{\alpha}{\sqrt{2}}(a+a^+)\right\rangle$$

$$= \frac{i}{\hbar} - m\omega^2\sqrt{\frac{\hbar}{m\omega}}\frac{1}{\sqrt{2}}\frac{1}{3}\left[\langle 0|e^{\frac{i\omega t}{2}} + \langle 1|e^{\frac{3i\omega t}{2}} + \langle 3|e^{\frac{7i\omega t}{2}}\right] (a+a^+)\left[|0\rangle e^{-\frac{i\omega t}{2}} + |1\rangle e^{-\frac{3i\omega t}{2}} + |3\rangle e^{-\frac{7i\omega t}{2}}\right]$$

$$= -\frac{1}{3}\sqrt{\hbar m\omega}\,\frac{\omega}{\sqrt{2}}\underbrace{\left[\langle 0|a|1\rangle e^{-i\omega t} + \langle 1|a^+|0\rangle e^{i\omega t}\right]}_{2\cos(\omega t)}$$

$$= -\frac{\sqrt{2}}{3}\sqrt{\hbar m\omega}\,\omega\cos(\omega t)$$

$$\frac{dE_\lambda}{d\lambda} = \frac{d\langle \psi_\lambda | H_\lambda | \psi_\lambda\rangle}{d\lambda} =$$

$$= \left\langle \frac{d\psi_\lambda}{d\lambda}\right| H_\lambda |\psi_\lambda\rangle + \left\langle \psi_\lambda \right| \frac{dH_\lambda}{d\lambda}\left|\psi_\lambda\right\rangle + \left\langle \psi_\lambda \right| H_\lambda \left| \frac{d\psi_\lambda}{d\lambda}\right\rangle$$

vl. stav

$$\left\langle \frac{d\psi_\lambda}{d\lambda}\right| E_\lambda |\psi_\lambda\rangle \qquad E_\lambda\left\langle \psi_\lambda \middle| \frac{d\psi_\lambda}{d\lambda}\right\rangle$$

$$E_\lambda \frac{d}{d\lambda}\langle \psi_\lambda|\psi_\lambda\rangle = 0$$

$$= \left\langle \psi_\lambda \right| \frac{dH_\lambda}{d\lambda}\left| \psi_\lambda\right\rangle$$

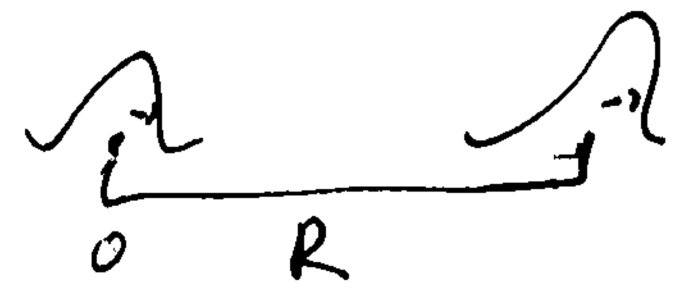
$$|0\rangle = \frac{1}{\sqrt{\alpha\sqrt{\pi}}}e^{-\frac{x^2}{2\alpha^2}} \qquad \alpha = \sqrt{\frac{\hbar}{m\omega}} \qquad \frac{d\cancel{|0\rangle}}{d\lambda} =$$

$$\frac{d|0\rangle}{d\omega} = \frac{d}{d\omega}\left[\frac{1}{\pi^{1/4}}\left(\frac{m\omega}{\hbar}\right)^{1/4} e^{-\frac{x^2 m\omega}{2\hbar}}\right] = \left(\frac{m}{\pi\hbar}\right)^{1/4}\left[\frac{1}{4}\omega^{-3/4} - \frac{x^2 m}{2\hbar}\omega^{1/4}\right]e^{-\frac{x^2 m\omega}{2\hbar}}$$

sand pro $m = \hbar = 1$

ÚKM 2021
IX - 1

- Mějme 2 LHO ve vzdálenosti R



- částice rozlišitelné

$|\psi_1^0\rangle = \cancel{\frac{1}{\sqrt{\alpha\sqrt{\pi}}}}\, |0\rangle_1 \qquad |\psi_2^0\rangle = |0\rangle_2$

Vlnová fce celku

$$|\psi\rangle = |0\rangle_1 \otimes |0\rangle_2 = |00\rangle$$
$$= \frac{1}{\sqrt{\alpha\sqrt{\pi}}} e^{-\frac{x_1^2}{2\alpha^2}} \cdot \frac{1}{\sqrt{\alpha\sqrt{\pi}}} e^{-\frac{(x_2-R)^2}{2\alpha^2}}$$

- Energie jednotlivé & celková

$$E_1 = \hbar\omega(n_1 + 1/2) \qquad E_2 = \hbar\omega(n_2 + 1/2)$$

$$E_{tot} = \hbar\omega(n_1 + n_2 + 1)$$
$$= \hbar\omega(a^+a \otimes 1 + 1 \otimes b^+b + 1)$$

- a, a^+ operátory:

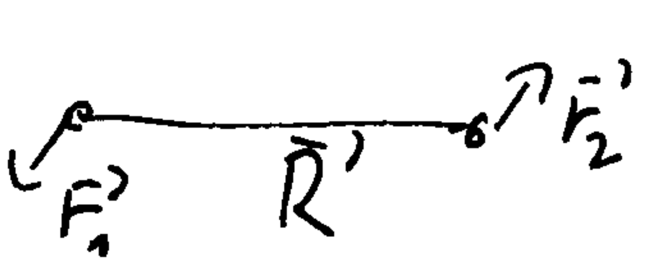
$1: \ a, a^+ \qquad 2: \ b, b^+ \qquad , \ x_1 = \frac{\alpha}{\sqrt{2}}(a + a^+) \qquad x_2 = \frac{\alpha}{\sqrt{2}}(b + b^+)$

tot $\quad a \otimes 1, \ a^+ \otimes 1 \qquad 1 \otimes b, \ 1 \otimes b^+$

působí $\quad a \otimes 1 |00\rangle = 0 \quad , \ a^+ \otimes 1 |00\rangle = \sqrt{1}\, |10\rangle$

Uvažujme, že částice jsou nabité a "připojené pružinkou" k jádru v 0, resp. R (Drudeho model)

3D:



$$V = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{|R|} + \frac{1}{|R - r_1 + r_2|} - \frac{1}{|R - r_1|} - \frac{1}{|R - r_2|}\right)$$

approx $|r_1|, |r_2| \ll R \qquad (|R - r_1| = \sqrt{(R_x - r_x)^2 + (R_y - r_y)^2 + (R_z - r_z)^2}$

Taylor $\left(1 - \frac{Rr}{R^2}\right)^{-1/2}$

$$V = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{\vec{r}_1 \cdot \vec{r}_2}{R^3} - \frac{3(\vec{R}\cdot\vec{r}_1)(\vec{R}\cdot\vec{r}_2)}{R^5}\right)$$

- Uvažujme $\vec{R} = (R, 0, 0)$

$$\to V = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{\vec{r}_1 \cdot \vec{r}_2}{R^3} - 3\frac{R r_1^x R r_2^x}{R^5}\right)$$
$$= \frac{e^2}{4\pi\varepsilon_0}\left(\frac{x_1x_2 + y_1y_2 + z_1z_2}{R^3} - \frac{3x_1x_2}{R^3}\right)$$
$$= \frac{e^2}{4\pi\varepsilon_0}\left(-\frac{2x_1x_2}{R^3} + \frac{y_1y_2 + z_1z_2}{R^3}\right)$$

UKM 2021
IX - 2

~~$V = \frac{e^2}{}$~~ Uvažujme jen x

$$V = \frac{e^2}{4\pi\varepsilon_0}\left(-\frac{2x_1x_2}{R^3}\right)$$

zapsat pomocí a, a^+, b, b^+

$$= -\frac{2e^2}{4\pi\varepsilon_0 R^3}(a+a^+)\otimes(b+b^+)\frac{\alpha^2}{2}$$

$$= -\frac{e^2\alpha^2}{4\pi\varepsilon_0 R^3}\cancel{(a+a^+)}(a\otimes b + a\otimes b^+ + a^+\otimes b + a^+\otimes b^+)$$

Maticové elementy se základním stavem

$a\otimes b\,|00\rangle = 0$

$a\otimes b^+|00\rangle = 0$

$a^+\otimes b\,|00\rangle = 0$

$a^+\otimes b^+|00\rangle = 1\,|11\rangle = |11\rangle$

$$\rightarrow \quad H = \begin{array}{c|cc} & |00\rangle & |11\rangle \\ \hline \langle 00| & \hbar\omega & -K \\ \langle 11| & -K & 3\hbar\omega \end{array} \qquad K = \frac{e^2\alpha^2}{4\pi\varepsilon_0 R^3}$$

$$\begin{vmatrix} \hbar\omega-\lambda & -K \\ -K & 3\hbar\omega-\lambda \end{vmatrix} = (\hbar\omega-\lambda)(3\hbar\omega-\lambda) - K^2 = \lambda^2 - 4\hbar\omega\lambda + 3\hbar^2\omega^2 - K^2$$

$$D = B^2 - 4AC = 16\hbar^2\omega^2 - 4\cdot(3\hbar^2\omega^2 - K^2) = 4\hbar^2\omega^2 + 4K^2$$

$$\lambda_{1,2} = \frac{-B\pm\sqrt{D}}{2A} = \frac{4\hbar\omega \pm\sqrt{4\hbar^2\omega^2+4K^2}}{2} = 2\hbar\omega \pm\sqrt{\hbar^2\omega^2+K^2}$$

$$= 2\hbar\omega \pm \hbar\omega\sqrt{1+\frac{K^2}{\hbar^2\omega^2}} \doteq 2\hbar\omega \pm \hbar\omega\left(1+\frac{1}{2}\frac{K^2}{\hbar^2\omega^2}\right)$$

$$\doteq \lambda_+ = 3\hbar\omega + \frac{1}{2}\frac{K^2}{\hbar\omega} \qquad \lambda_- = \hbar\omega - \frac{1}{2}\frac{K^2}{\hbar\omega} = \hbar\omega - \frac{1}{2}\frac{e^4\alpha^4}{(4\pi\varepsilon_0)^2 R^6}\frac{1}{\hbar\omega}$$

$$\alpha^4 = \left(\sqrt{\frac{\hbar}{m\omega}}\right)^4 = \frac{\hbar^2}{m^2\omega^2}$$

$$= \hbar\omega - \frac{1}{2}\frac{e^4}{(4\pi\varepsilon_0)^2}\frac{\hbar}{m^2\omega^3}\frac{1}{R^6}$$

$-\frac{1}{R^6} \sim$ vdW inter.

• LHO - pásma, vdW, x^2 čas vývoj?, a, a^+ na x-stav, a, a^+ na p-stav, OC, coma basics? x^3 v prvním order
HF theorem

UKM-2021
VII - 1

• Mějme $\psi = \frac{2}{\sqrt{3\alpha\sqrt{\pi}}}\frac{x^2}{\alpha^2}e^{-\frac{x^2}{2\alpha^2}}$, jaký je časový vývoj? pokud se nachází v potenciálu $\frac{1}{2}m\omega^2x^2$

• Normalizace:

$$\frac{3}{2B}\frac{1}{2B}\sqrt{\frac{\pi}{B}} \qquad B=\frac{1}{\alpha^2}$$

$$\int_{-\infty}^{\infty}\frac{4}{3\alpha\sqrt{\pi}}\frac{x^4}{\alpha^4}e^{-\frac{x^2}{\alpha^2}}dx = \frac{4}{3\alpha\sqrt{\pi}\,\alpha^4}\cdot\frac{3\alpha^2}{2}\frac{\alpha^2}{2}\sqrt{\pi}\alpha = 1 \quad \text{OK}$$

• ~~časový vývoj~~ koef.

$$\langle 0|\psi\rangle = \frac{1}{\sqrt{\alpha\sqrt{\pi}}}\frac{2}{\sqrt{3\alpha\sqrt{\pi}}}\int_{-\infty}^{\infty}\frac{x^2}{\alpha^2}e^{-\frac{x^2}{\alpha^2}}dx$$

$$|0\rangle = \frac{1}{\sqrt{\alpha\sqrt{\pi}}}e^{-\frac{x^2}{2\alpha^2}}$$

$$= \frac{2}{\alpha\sqrt{\pi}\sqrt{3}\cancel{\alpha^2}}\sqrt{\pi}\,\alpha\frac{\alpha^2}{2} = \frac{1}{\sqrt{3}} = c_0$$

$$c_2 = \sqrt{1-c_0^2} = \sqrt{\frac{2}{3}}$$ ... v principu $e^{i\delta}$, porovnáním s $|0\rangle$ zjistíme, že $\delta = 0$

$$\rightarrow |\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|2\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle e^{-\frac{i\omega t}{2}} + \sqrt{\frac{2}{3}}|2\rangle e^{-i\frac{5\omega t}{2}}$$

$$\rho = \psi^*\psi = \left[\frac{1}{\sqrt{3}}\frac{1}{\sqrt{\alpha\sqrt{\pi}}}e^{-\frac{x^2}{2\alpha^2}}e^{\frac{i\omega t}{2}} + \sqrt{\frac{2}{3}}\frac{1}{\sqrt{2\alpha\sqrt{\pi}}}\left[\frac{2x^2}{\alpha^2}-1\right]e^{-\frac{x^2}{2\alpha^2}}e^{\frac{i5\omega t}{2}}\right]$$

$$\left[\frac{1}{\sqrt{3}}\frac{1}{\sqrt{\alpha\sqrt{\pi}}}e^{-\frac{x^2}{2\alpha^2}}e^{-\frac{i\omega t}{2}} + \sqrt{\frac{2}{3}}\frac{1}{\sqrt{2\alpha\sqrt{\pi}}}\left[\frac{2x^2}{\alpha^2}-1\right]e^{-\frac{x^2}{2\alpha^2}}e^{-\frac{i5\omega t}{2}}\right]$$

$$= \frac{1}{\alpha\sqrt{\pi}}\frac{1}{3}e^{-\frac{x^2}{\alpha^2}} + \frac{2}{3}\frac{1}{2\alpha\sqrt{\pi}}e^{-\frac{x^2}{\alpha^2}}\left(\frac{2x^2}{\alpha^2}-1\right)^2$$

$$+\frac{\sqrt{2}}{3}\frac{1}{\alpha\sqrt{\pi}}\frac{1}{\sqrt{2}}e^{-\frac{x^2}{\alpha^2}}\underbrace{\left[e^{-2\omega t}+e^{2\omega t}\right]}_{2\cos(2\omega t)\,\sim\,\cos(\Delta E)}\left[\frac{2x^2}{\alpha^2}-1\right]$$