1 - @25.4.

2. spin

MKM2023

T7.5.-1

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & \hbar/2 \\ \hbar/2 & -\lambda \end{vmatrix} = \lambda^2 - \hbar^2/4 = (\lambda - \hbar/2)(\lambda + \frac{\hbar}{2}) \to \lambda = \pm \frac{\hbar}{2}$$

$$\lambda = +\frac{\hbar}{2} \to \begin{pmatrix} -\frac{\hbar}{2} & \frac{\hbar}{2} \\ \frac{\hbar}{2} & -\frac{\hbar}{2} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0 \to \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{ normalizace } 1^2 + 1^2 = 2$$

$$\to \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda = -\frac{\hbar}{2} \to \begin{pmatrix} \frac{\hbar}{2} & \frac{\hbar}{2} \\ \frac{\hbar}{2} & \frac{\hbar}{2} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0 \to \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\left(\text{nebo } \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \leftarrow \text{pouze fáze se liší}\right)$

2.2 $\hat{O} = \sum \lambda_i |i\rangle\langle i|$

$$\hat{S}_x = \cancel{\frac{\hbar}{2}\frac{1}{\sqrt{2}}(1\ 1)\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{\hbar}{2}\frac{1}{2}(1\ -1)\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{\hbar}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \frac{\hbar}{4}\Big(1}$$

$$\hat{S}_x = \frac{\hbar}{2}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}}(1\ 1) - \frac{\hbar}{2}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}(1\ -1)\frac{1}{\sqrt{2}} = \frac{\hbar}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \frac{\hbar}{4}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \frac{\hbar}{4}\begin{pmatrix} 1-1 & 1+1 \\ 1+1 & 1-1 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \leftarrow \text{báze vl. stavů } \hat{S}_z$$

2.3 $\hat{S}_x$, $S_x^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $S_x^- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}(1\ 0) - \frac{\hbar}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}(0\ 1) = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \leftarrow \text{v bázi vl. stavů } \hat{S}_x$$

2.4 $U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$U^+ S_x U = \frac{1}{2}\frac{\hbar}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{\hbar}{4}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{\hbar}{4}\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

OK

UKM 2023
T2.5. -2

3.1

$$\langle Y_0^0 | \frac{x}{r} | Y_1^m \rangle$$

$$\langle Y_0^0 | \frac{x}{r} = \frac{1}{\sqrt{4\pi}} \sin\theta \cos\phi = \frac{1}{\sqrt{4\pi}} \sin\theta \frac{1}{2} [e^{i\phi} + e^{-i\phi}]$$

$$= \frac{1}{4} \frac{1}{\sqrt{\pi}} (\sin\theta e^{i\phi} + \sin\theta e^{-i\phi}) = \frac{1}{\sqrt{6}} \sqrt{\frac{3}{8\pi}} \sin\theta ((e^{i\phi}) + e^{-i\phi})$$

$$= \frac{1}{\sqrt{6}} (-Y_1^{+1} + Y_1^{-1})$$

3.2

$$\langle Y_0^0 | \frac{x}{r} | Y_1^1 \rangle = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \frac{1}{\sqrt{4\pi}} \sin\theta \cos\phi (-\sin\theta e^{i\phi}) \sqrt{\frac{3}{8\pi}}$$

$$= \underbrace{\int_0^{2\pi} d\phi \cos\phi e^{i\phi}}_{\frac{1}{2} 2\pi} \int_0^{\pi} d\theta \frac{1}{\sqrt{4\pi}} \sin^3\theta \sqrt{\frac{3}{8\pi}} \cdot (-1)$$

← znaménko z Y_1^1

znaménko (-) ze substituce

$$= + \frac{\pi}{\sqrt{4\pi}} \sqrt{\frac{3}{8\pi}} \int_1^{-1} (1-t^2) dt = + \frac{\sqrt{3}}{4\sqrt{2}} \left[t - \frac{t^3}{3} \right]_1^{-1} = + \frac{\sqrt{3}}{4\sqrt{2}} \left[-1 - 1 + \frac{1}{3} + \frac{1}{3} \right] = + \frac{\sqrt{3}}{4\sqrt{2}} \left(-\frac{4}{3}\right) = \frac{1}{\sqrt{6}}$$ OK

$$\langle Y_0^0 | \frac{x}{r} | Y_1^0 \rangle = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \frac{1}{\sqrt{4\pi}} \sin\theta \cos\phi \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$= \underbrace{\int_0^{2\pi} d\phi \cos\phi}_{= 0} \int_0^{\pi} d\theta \ldots = 0$$

$$\langle Y_0^0 | \frac{x}{r} | Y_1^{-1} \rangle = \ldots + \frac{1}{\sqrt{6}}$$ OK

3.3

$$\langle Y_0^0 | \frac{z}{r} \ldots = \frac{1}{\sqrt{4\pi}} \cos\theta = \frac{1}{\sqrt{3}} \sqrt{\frac{3}{4\pi}} \cos\theta = \frac{1}{\sqrt{3}} \langle Y_1^0 |$$