plan oscill in e field
ii) udW

Ukol-2023
T-11.4-1

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$$

$$V = -|e|\hat{x}E$$

1.1. $\frac{1}{2}m\omega^2 x^2 - exE = \frac{1}{2}m\omega^2\left(x^2 - \frac{2exE}{m\omega^2}\right) = \frac{1}{2}m\omega^2\left(x - \frac{eE}{m\omega^2}\right)^2 + \underbrace{\frac{1}{2}m\omega^2 \frac{e^2E^2}{m^2\omega^4}}_{= \frac{1}{2}\frac{e^2E^2}{m\omega^2}}$

$A^2 \quad -2AB \qquad B = \frac{eE}{m\omega^2}$

$$\underline{\underline{= \frac{1}{2}\frac{e^2E^2}{m\omega^2}}}$$

1.2. $H_0 = \begin{pmatrix} \frac{1}{2}\hbar\omega & & \\ & \frac{3}{2}\hbar\omega & \\ & & \frac{5}{2}\hbar\omega \end{pmatrix}$ $\quad V = -|e|\hat{x}E = -|e|E\frac{d}{\sqrt{2}}(a+a^+) = -\frac{eE}{\sqrt{2}}\sqrt{\frac{\hbar}{m\omega}}(a+a^+)$

$$= \underbrace{-\frac{eE}{\sqrt{2}}\sqrt{\frac{\hbar}{m\omega}}}_{A}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

$$H = \begin{pmatrix} \frac{1}{2}\hbar\omega & -A & 0 \\ -A & \frac{3}{2}\hbar\omega & -A \\ 0 & -A & \frac{5}{2}\hbar\omega \end{pmatrix} \qquad H_{2\times 2} = \begin{pmatrix} \frac{1}{2}\hbar\omega & -A \\ -A & \frac{3}{2}\hbar\omega \end{pmatrix}$$

$$\text{diag } H_{2\times 2}: \quad (H - \lambda\mathbb{1}) = \begin{pmatrix} \frac{1}{2}\hbar\omega - \lambda & -A \\ -A & \frac{3}{2}\hbar\omega - \lambda \end{pmatrix}$$

$$Det = \left(\frac{1}{2}\hbar\omega - \lambda\right)\left(\frac{3}{2}\hbar\omega - \lambda\right) - A^2 = \frac{3}{4}\hbar^2\omega^2 - 2\hbar\omega\lambda + \lambda^2 - A^2$$

$$= \lambda^2 - 2\hbar\omega\lambda + \frac{3}{4}\hbar^2\omega^2 - A^2$$

$$Dis: \quad 4\hbar^2\omega^2 - 4\left(\frac{3}{4}\hbar^2\omega^2 - A^2\right) = 4\hbar^2\omega^2 - 3\hbar^2\omega^2 + 4A^2 = \hbar^2\omega^2 + 4A^2$$

$$\lambda_{1,2} = \frac{2\hbar\omega \pm \sqrt{\hbar^2\omega^2 + 4A^2}}{2} = \hbar\omega \pm \frac{1}{2}\underbrace{\sqrt{\hbar^2\omega^2 + 4A^2}}_{\text{assume } A^2 \ll \hbar^2\omega^2}$$

$$\sqrt{1+x} \approx 1 + \frac{x}{2} - \frac{x^2}{8}$$

$$= \hbar\omega \pm \frac{1}{2}\sqrt{\hbar^2\omega^2\left(1 + \frac{4A^2}{\hbar^2\omega^2}\right)} = \hbar\omega \pm \frac{1}{2}\hbar\omega\sqrt{1 + \frac{4A^2}{\hbar^2\omega^2}}$$

$$\approx \hbar\omega \pm \frac{1}{2}\hbar\omega\left[1 + \frac{4A^2}{2\hbar^2\omega^2} - \frac{16A^4}{8\hbar^4\omega^4}\right]$$

základní stav: $\lambda_- = \hbar\omega - \frac{1}{2}\hbar\omega - \frac{4A^2}{2\hbar^2\omega^2}\frac{1}{2}\hbar\omega - \frac{1}{2}\hbar\omega\frac{16A^4}{8\hbar^4\omega^4}$

$$= \frac{1}{2}\hbar\omega - \frac{A^2}{\hbar\omega} - \frac{A^4}{\hbar^3\omega^3} \qquad A^2 = \frac{e^2E^2d^2}{2} = \frac{e^2E^2\hbar}{2m\omega}$$

$$= \frac{1}{2}\hbar\omega - \frac{e^2E^2\hbar}{2m\omega\hbar\omega} - \frac{e^4E^4\hbar^2}{4m^2\omega^2}\frac{1}{\hbar^3\omega^3} = \frac{1}{2}\hbar\omega - \underline{\underline{\frac{e^2E^2}{2m\omega^2}}} - \frac{e^4E^4}{4m^2\hbar\omega^5}$$

1.3. $\psi(x, x_0) = \frac{1}{\sqrt{\alpha\sqrt{\pi}}} e^{-\frac{(x-x_0)^2}{2\alpha^2}}$

UKM. 2023
T-14.4.-2.

$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 - |e| x E$

$\langle |p^2| \rangle = \frac{1}{4}\hbar\omega$ ← nezávisí na x_0

$$\frac{1}{\alpha\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-\frac{(x-x_0)^2}{\alpha^2}} \frac{1}{2} m\omega^2 x^2 \, dx = \begin{matrix} t = x - x_0 \\ dt = dx \end{matrix} = \frac{1}{\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{\alpha^2}} \frac{1}{2} m\omega^2 (t + x_0)^2 dt$$

$$= \frac{1}{\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{\alpha^2}} \frac{1}{2} m\omega^2 (t^2 + \cancel{2 t x_0} + x_0^2) dt$$

$$= \frac{1}{\alpha\sqrt{\pi}} \frac{1}{2} m\omega^2 \int_{-\infty}^{\infty} e^{-\frac{t^2}{\alpha^2}} (\underset{\frac{\alpha^3\sqrt{\pi}}{2}}{t^2} + \underset{\alpha\sqrt{\pi}}{x_0^2}) dt = \frac{1}{2} m\omega^2 \left(\frac{\alpha^2}{2} + x_0^2\right) =$$

$$= \underbrace{\frac{1}{2} m\omega^2 \frac{1}{2} \frac{\hbar}{m\omega}}_{\frac{1}{4}\hbar\omega} + \frac{1}{2} m\omega^2 x_0^2$$

$$\langle -|e| x E \rangle = \frac{1}{\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-x_0)^2}{\alpha^2}} (-|e| x E) dx = \begin{matrix} t = x - x_0 \\ dt = dx \end{matrix} = \frac{1}{\alpha\sqrt{\pi}} \int_{-\infty}^{\infty} e^{\frac{-(x-x_0)^2}{\alpha^2}} (-|e| (\cancel{t} + \underset{x_0 \alpha\sqrt{\pi}}{x_0}) E) dt$$

$$= -x_0 |e| E$$

$$\langle H \rangle_{x_0} = \frac{1}{2}\hbar\omega + \frac{1}{2} m\omega^2 x_0^2 - x_0 |e| E$$

$$\frac{dE}{dx_0} = m\omega^2 x_0 - |e| E = 0$$

$$x_0 = \frac{|e| E}{m\omega^2}$$

$$\langle H \rangle = \frac{1}{2}\hbar\omega + \frac{1}{2} m\omega^2 \frac{|e|^2 E^2}{m^2\omega^4} - \frac{|e|^2 E^2}{m\omega^2}$$

$$= \frac{1}{2}\hbar\omega - \frac{1}{2} \frac{|e|^2 E^2}{m\omega^2}$$

1.4.

$$|f\rangle = \cancel{\frac{1}{\sqrt{1+c^2}} (|0\rangle + c)} = \frac{1}{\sqrt{1+c^2}} (|0\rangle + c|1\rangle)$$

$$\langle f | H | f \rangle = \frac{1}{1+c^2} (\langle 0| + c\langle 1|)(a^+ a + \tfrac{1}{2})\hbar\omega (|0\rangle + c|1\rangle) + \frac{1}{1+c^2} (\langle 0| + c\langle 1|)$$

$$-|e| E \frac{\alpha}{\sqrt{2}} (a + a^+)(|0\rangle + c|1\rangle) =$$

$$= \frac{1}{1+c^2} \left[\frac{1}{2}\hbar\omega + c^2 \frac{3}{2}\hbar\omega\right] + \frac{1}{1+c^2} \left(-|e| E \frac{\alpha}{\sqrt{2}}\right)(c + c)$$

$$= \frac{1}{2}\hbar\omega + \frac{c^2}{1+c^2}\hbar\omega + \frac{2c}{1+c^2} \left(-|e| E \frac{\alpha}{\sqrt{2}}\right)$$

$$\frac{dE(c)}{dc} = \frac{2c\hbar\omega}{1+c^2} - \frac{c^2\hbar\omega \, 2c}{(1+c^2)^2} + \frac{2(-|e| E \frac{\alpha}{\sqrt{2}})}{1+c^2} - \frac{2c(-|e| E \frac{\alpha}{\sqrt{2}})}{(1+c^2)^2} 2c = 0$$

$$\frac{2\hbar\omega c[1 + c^2 - c^2]}{(1+c^2)^2} - \frac{|e| E \frac{\alpha}{\sqrt{2}} 2[1 + c^2 - 2c^2]}{(1+c^2)^2} = 0$$

...graf...

UKM - 2023
T - 18.4. - 3

2.1. $\hat{H} = \hat{H}_1 \otimes 1 + 1 \otimes \hat{H}_2$

$$H_1 |n_1\rangle = \hbar\omega\left(a^+a + \tfrac{1}{2}\right)|n_1\rangle = \hbar\omega\left(n + \tfrac{1}{2}\right)|n_1\rangle$$

$|n_1\rangle \otimes |n_2\rangle \rightarrow |n_1 n_2\rangle$ nová báze

$$\langle n_1| \otimes \langle n_2| \left(H_1 \otimes 1 + 1 \otimes H_2\right)|n_1\rangle \otimes |n_2\rangle = \langle n_1| \otimes \langle n_2| H_1 \otimes 1 |n_1\rangle \otimes |n_2\rangle$$
$$+ \langle n_1| \otimes \langle n_2| 1 \otimes H_2 |n_1\rangle \otimes |n_2\rangle = \hbar\omega\left(n_1 + \tfrac{1}{2}\right) + \hbar\omega\left(n_2 + \tfrac{1}{2}\right)$$
$$= \hbar\omega\left(n_1 + n_2 + 1\right)$$

2.2. $\hat{H} = \hbar\omega\left(a^+a + \tfrac{1}{2}\right) \otimes 1 + \hbar\omega\, 1 \otimes \left(b^+b + \tfrac{1}{2}\right)$

$$\langle n_1 n_2| \hbar\omega\left(a^+a + \tfrac{1}{2}\right)\otimes 1 + \hbar\omega\, 1 \otimes \left(b^+b + \tfrac{1}{2}\right)|n_1 n_2\rangle$$
$$= \hbar\omega \underbrace{\langle n_1| a^+a + \tfrac{1}{2}|n_1\rangle}_{n_1 + \frac{1}{2}} \otimes \underbrace{\langle n_2|1|n_2\rangle}_{1} + \hbar\omega\left(n_2 + \tfrac{1}{2}\right) = \hbar\omega\left(n_1 + n_2 + 1\right)$$

1D:

$$V = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{R} + \frac{1}{R - x_1 + x_2} - \frac{1}{R - x_1} - \frac{1}{R + x_2}\right)$$

$$\frac{1}{1+x} \approx 1 - x + x^2 - x^3$$

$$\frac{1}{1-x} \approx 1 + x + x^2 + x^3$$

$$= \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{R} + \frac{1}{R\left(1 - \frac{x_1 + x_2}{R}\right)} - \frac{1}{R\left(1 - \frac{x_1}{R}\right)} - \frac{1}{R\left(1 + \frac{x_2}{R}\right)}\right]$$

$$\doteq \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{R} + \frac{1}{R}\left(1 + \frac{x_1 + x_2}{R} + \frac{(x_1 + x_2)^2}{R^2}\right) - \frac{1}{R}\left(1 + \frac{x_1}{R} + \frac{x_1^2}{R^2}\right) - \frac{1}{R}\left(1 - \frac{x_2}{R} + \frac{x_2^2}{R^2}\right)\right]$$

$$= \frac{e^2}{4\pi\varepsilon_0}\left[\cancel{\frac{1}{R}} + \cancel{\frac{1}{R}} + \frac{x_1 + x_2}{R^2} + \frac{x_1^2 + 2x_1x_2 + x_2^2}{R^3} - \cancel{\frac{1}{R}} - \frac{x_1}{R^2} - \frac{x_1^2}{R^3} - \cancel{\frac{1}{R}} + \frac{x_2}{R^2} - \frac{x_2^2}{R^3}\right]$$

$$= \frac{e^2}{4\pi\varepsilon_0}\left(-\frac{2x_1x_2}{R^3}\right) = V_{1D}$$

2.3 $$\hat{V}_{1D} = -\frac{e^2}{4\pi\varepsilon_0}\frac{2\hat{x}_1\hat{x}_2}{R^3} = -\frac{2e^2}{4\pi\varepsilon_0}\frac{\alpha^2}{2}\frac{(a^+ + a)\otimes(b^+ + b)}{R^3}$$

2.4 $$\hat{V}_{1D}|00\rangle = -\frac{e^2\alpha^2}{4\pi\varepsilon_0}\frac{(a^+ + a)\otimes(b^+ + b)}{R^3}|00\rangle = -\frac{e^2\alpha^2}{4\pi\varepsilon_0 R^3}\left(a^+b^+ + a^+b + ab^+ + ab\right)|00\rangle$$
$$= -\frac{e^2\alpha^2}{4\pi\varepsilon_0 R^3}\, a^+b^+|00\rangle = -\frac{e^2\alpha^2}{4\pi\varepsilon_0 R^3}|11\rangle$$

2.5. - stačí v bázi $|00\rangle, |11\rangle$

$$H = \begin{pmatrix} \hbar\omega & -A \\ -A & 3\hbar\omega \end{pmatrix}$$

UKM-2023
T-18.4.-4

$$H = \begin{pmatrix} \hbar\omega & -A \\ -A & 3\hbar\omega \end{pmatrix} \qquad A = \frac{e^2 d^2}{4\pi\varepsilon_0 R^3}$$

$$|H - \lambda \mathbb{1}| = \begin{vmatrix} \hbar\omega - \lambda & -A \\ -A & 3\hbar\omega - \lambda \end{vmatrix} = (\hbar\omega - \lambda)(3\hbar\omega - \lambda) - A^2$$

$$= \lambda^2 - 4\hbar\omega\lambda + 3\hbar^2\omega^2 - A^2$$ ← "same" as before...

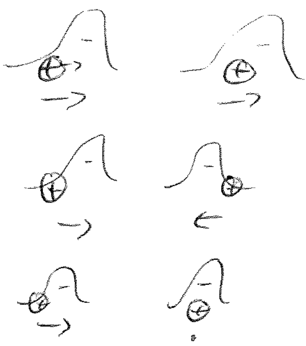
$$D = 16\hbar^2\omega^2 - 12\hbar^2\omega^2 + 4A^2 = 4\hbar^2\omega^2 + 4A^2$$

$$\lambda_{1,2} = \frac{4\hbar\omega \pm \sqrt{4\hbar^2\omega^2 + 4A^2}}{2}$$ opět $4\hbar^2\omega^2 \gg 4A^2$ ← porucha

$$= \frac{4\hbar\omega \pm 2\hbar\omega\sqrt{1 + \frac{A^2}{\hbar^2\omega^2}}}{2} = 2\hbar\omega \pm \hbar\omega\left(1 + \frac{A^2}{2\hbar^2\omega^2}\right) =$$

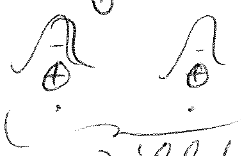
$$\lambda = \hbar\omega - \frac{A^2}{2\hbar\omega} = \hbar\omega - \frac{1}{2\hbar\omega}\frac{e^4 d^4}{(4\pi\varepsilon_0)^2 R^6} = \hbar\omega - \frac{e^4}{(4\pi\varepsilon_0)^2 R^6}\frac{\hbar^2}{m\omega}\frac{1}{2\hbar\omega}$$

$$= \hbar\omega - \frac{e^4}{(4\pi\varepsilon_0)^2 R^6}\frac{\hbar}{2m\omega^2}$$ ← snížení E základního stavu korelací částic



← stav má nižší energii než

tyto stavy



základní stav neinteragujících částic