21.7.3 obecný tvar řešení Š. rovnice 

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$ ← Š. rovnice

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{p^2}{2m}\psi$$ ← pro volnou částici

$$\psi(x,t) = \varphi(x)\,\chi(t)$$

$$\varphi(x)\, i\hbar \frac{\partial \chi(t)}{\partial t} = \chi(t)\left(-\frac{\hbar^2}{2m}\right)\frac{\partial^2}{\partial x^2}\varphi(x)$$

$$\frac{i\hbar}{\chi(t)}\frac{\partial \chi(t)}{\partial t} = -\frac{\hbar^2}{2m\varphi(x)}\frac{\partial^2 \varphi(x)}{\partial x^2} = \text{konstanta} = E$$

$\varphi(x) = e^{ikx}$ (uhodneme)

$$\frac{\hbar^2 k^2}{2m} = E$$

$$\frac{i\hbar\, \partial \chi(t)}{\chi(t)\,\partial t} = \frac{\hbar^2 k^2}{2m}$$

$\chi(t) = e^{ct}$ (uhodneme)

$$i\hbar c = \frac{\hbar^2 k^2}{2m}$$

$$c = -\frac{i\hbar k^2}{2m} = -\frac{iE}{\hbar}$$

$$\rightarrow \psi(x,t) = e^{ikx - \frac{iEt}{\hbar}} = e^{ikx - \frac{i\hbar k^2 t}{2m}}$$ pro dané $k$ (nějaké)

počáteční podmínka $\psi(x,0) = \int c_k e^{ikx}\,dk$

$$\psi(x,t) = \int c_k\, e^{ikx - \frac{i\hbar k^2 t}{2m}}$$

hustota toku pravděpodobnosti pro

$$\psi = A e^{(Et - \sqrt{2mE}\,x)/(i\hbar)}$$

$$j = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right)$$

$$= \frac{\hbar}{2mi}\Big[A e^{-(Et - \sqrt{2mE}\,x)/(i\hbar)} \left(-\frac{\sqrt{2mE}}{i\hbar}\right) A e^{(Et - \sqrt{2mE}\,x)/(i\hbar)}$$

$$- A e^{(Et - \sqrt{2mE}\,x)/(i\hbar)} \left(\frac{\sqrt{2mE}}{i\hbar}\right) A e^{-(Et - \sqrt{2mE}\,x)/(i\hbar)}\Big]$$

$$= \frac{\hbar}{2mi}\left[A^2 \cdot 2 \cdot \left(- \frac{\sqrt{2mE}}{i\hbar}\right)\right] = \frac{\underbrace{A^2}_{|\psi|^2} \underbrace{\sqrt{2mE}}_{\frac{p\psi}{\psi}}}{m} \rightarrow \text{"}\frac{p}{m}|\psi|^2\text{"} = v|\psi|^2$$

$$\psi = A \exp\left[(Et - \sqrt{2mE}\,x)/(i\hbar)\right] + B \exp\left[(Et + \sqrt{2mE}\,x)/(i\hbar)\right]$$

$$\rightarrow A^2 \sqrt{\frac{2E}{m}} - B^2 \sqrt{\frac{2E}{m}} = \sqrt{\frac{2E}{m}}\left(A^2 - B^2\right) \qquad \text{OK}$$

21.7. 6 –

UKMT4.4

$$\int_{-\infty}^{\infty} \exp(-\alpha x^2) = \sqrt{\frac{\pi}{\alpha}}$$

$t = 0$ $\quad \psi(x,0) = \cancel{N \exp\left[-x^2/2a^2 + i k_0 x\right]}$

$$= N \exp\left[-\frac{x^2}{2a^2} + i k_0 x\right]$$

$a, k \in \mathbb{R}$, určete $N$ a $\vec{j}$

$$1 = \langle \psi | \psi \rangle = N^2 \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{2a^2} - i k_0 x\right] \exp\left[-\frac{x^2}{2a^2} + i k_0 x\right] dx$$

$$= N^2 \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{a^2}\right] dx = N^2 \sqrt{\frac{\pi}{\frac{1}{a^2}}} = N^2 a\sqrt{\pi} = 1$$

$$N = \frac{1}{\sqrt{a\sqrt{\pi}}}$$

OK

hustota r.p.

$$\rho = \psi^* \psi = \frac{1}{a\sqrt{\pi}} \exp\left[-\frac{x^2}{2a^2} - i k_0 x\right] \exp\left[-\frac{x^2}{2a^2} + i k_0 x\right]$$

$$= \frac{1}{a\sqrt{\pi}} \exp\left[-\frac{x^2}{a^2}\right]$$

hustota toku r.p.

$$j = \frac{\hbar}{2mi}\left[\psi^* \frac{\partial \psi}{dx} - \psi \frac{\partial \psi^*}{\partial x}\right] = \frac{\hbar}{2mi}\left[\left(-\frac{2x}{2a^2} + i k_0\right)\psi^*\psi - \left(-\frac{2x}{2a^2} - i k_0\right)\psi\psi^*\right]$$

$$= \frac{\hbar k_0}{m} \rho(x,0)$$

7) FT($\psi$)

$$\psi(k,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x,0) e^{-ikx} dx = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{a\sqrt{\pi}}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{2a^2} + i k_0 x\right] \exp[-ikx] dx$$

$$= \frac{1}{\sqrt{2\pi a\sqrt{\pi}}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{2a^2} + i(k_0 - k)x\right] dx = *$$

$$-\left[\frac{x^2}{2a^2} - i(k_0 - k)x\right] = -\left[\frac{x^2}{2a^2} - 2\frac{1}{a\sqrt{2}} \frac{ia(k_0-k)}{\sqrt{2}} x - \frac{a^2(k_0-k)^2}{2}\right] - \frac{a^2(k_0-k)^2}{2}$$

$$2AB = i(k_0 - k)$$

$\overset{\shortparallel}{A^2 x^2} \quad \overset{\shortparallel}{-2ABx}$

$(Ax - B)^2$

$$A^2 = \frac{1}{2a^2} \qquad A = \frac{1}{a\sqrt{2}}$$

$$2\frac{1}{a\sqrt{2}} B = i(k_0 - k)$$

$$B = \frac{ia(k_0-k)}{\sqrt{2}} \qquad B^2 = -\frac{a^2(k_0-k)^2}{2}$$

$$* = \frac{1}{\sqrt{2\pi a\sqrt{\pi}}} \int_{-\infty}^{\infty} \exp\left[-\left(\frac{x}{a\sqrt{2}} - \frac{ia(k_0-k)}{\sqrt{2}}\right)^2\right] \exp\left[-\frac{a^2(k_0-k)^2}{2}\right] dx$$

$$\frac{x}{a\sqrt{2}} - \frac{ia(k_0-k)}{\sqrt{2}} = \frac{t}{a\sqrt{2}}$$

$$dx = dt$$

$$= \frac{1}{\sqrt{2\pi a\sqrt{\pi}}} \int_{-\infty}^{\infty} \exp\left[-\frac{t^2}{2a^2}\right] \exp\left[-a^2(k_0-k)^2/2\right] dt$$

$$\sqrt{\frac{\pi}{\frac{1}{2a^2}}} = a\sqrt{2\pi}$$

$$= \sqrt{\frac{a}{\sqrt{\pi}}} \exp\left[-a^2(k_0-k)^2/2\right]$$

← Gaussovský balík okolo $k_0$ (normalisovaný !?)

8) $\langle\hat{x}\rangle$ , $\langle\hat{p}\rangle$ $\qquad \psi = \frac{1}{\sqrt{a\sqrt{\pi}}} \exp\left[-\frac{x^2}{2a^2} + ik_0 x\right]$ UKMT3.45

$$\langle x\rangle = \langle\psi|x|\psi\rangle = \frac{1}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{2a^2} - ik_0x\right] x \exp\left[-\frac{x^2}{2a^2} + ik_0x\right]$$

$$= \frac{1}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{a^2}\right] x \, dx = 0 \quad \text{OK}$$

$$\langle p\rangle = \langle\psi|\hat{p}|\psi\rangle = -i\hbar \langle\psi|\frac{d}{dx}|\psi\rangle =$$

$$= -i\hbar \frac{1}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{2a^2} - ik_0x\right] \frac{d}{dx} \exp\left[-\frac{x^2}{2a^2} + ik_0x\right] dx$$

$$= -i\hbar \frac{1}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{2a^2} - ik_0x\right] \left(-\frac{x}{a^2} + ik_0\right) \exp\left[-\frac{x^2}{2a^2} + ik_0x\right] dx$$

$$= \frac{1}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{a^2}\right] \left(\underbrace{\frac{i\hbar x}{a^2}}_{=0} + \hbar k_0\right) dx$$

$$= \frac{1}{a\sqrt{\pi}} \hbar k_0 \sqrt{\frac{\pi}{\frac{1}{a^2}}} = \hbar k_0 \quad \text{OK}$$

9) $\langle(\Delta x)^2\rangle$ ; $\langle(\Delta p)^2\rangle$ $\qquad \int_{-\infty}^{\infty} x^2 \exp[-\alpha x^2] = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}}$

$\Delta x = x - \underbrace{\langle x\rangle}_{=0} \Rightarrow \Delta x = x \qquad (\Delta x)^2 = x^2 \qquad \langle(\Delta x)^2\rangle = \langle x^2\rangle$

$$\langle x^2\rangle = \langle\psi|x^2|\psi\rangle = \frac{1}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{2a^2} - ik_0x\right] x^2 \exp\left[-\frac{x^2}{2a^2} + ik_0x\right] dx$$

$$= \frac{1}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{a^2}\right] x^2 \, dx = \frac{1}{a\sqrt{\pi}} \frac{a^2}{2} \sqrt{a^2\pi} = \frac{a^2}{2} \quad \text{OK}$$

$$\langle(\Delta p)^2\rangle = \left\langle\left(-i\hbar\frac{d}{dx}\right)^2\right\rangle = -\hbar^2 \left\langle\frac{d^2}{dx^2}\right\rangle$$

$$-\hbar^2\left\langle\frac{d^2}{dx^2}\right\rangle = -\hbar^2 \langle\psi|\frac{d^2}{dx^2}|\psi\rangle = -\frac{\hbar^2}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{2a^2} - ik_0x\right] \frac{d^2}{dx^2} \exp\left[-\frac{x^2}{2a^2} + ik_0x\right]$$

$$\frac{d}{dx}\frac{d}{dx} \exp\left[-\frac{x^2}{2a^2} + ik_0x\right] = \frac{d}{dx}\left(-\frac{x}{a^2} + ik_0\right) \exp\left[-\frac{x^2}{2a^2} + ik_0x\right] =$$

$$= \left[-\frac{1}{a^2} - \frac{x}{a^2}\left(-\frac{x}{a^2} + ik_0\right) + ik_0\left(-\frac{x}{a^2} + ik_0\right)\right] \exp\left(-\frac{x^2}{2a^2} + ik_0x\right)$$

$$= \left[-\frac{1}{a^2} + \left(-\frac{x}{a^2} + ik_0\right)^2\right] \exp\left(-\frac{x^2}{2a^2} + ik_0x\right)$$

$$\langle (p)^2 \rangle = -\frac{\hbar^2}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{a^2}\right] \left[-\frac{1}{a^2} + \left(-\frac{x}{a^2} + ik_0\right)^2\right] dx$$

$$= -\frac{\hbar^2}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{x^2}{a^2}\right] \left[-\frac{1}{a^2} + \frac{x^2}{a^4} - \underbrace{\frac{2ik_0x}{a^2}}_{=0} - k_0^2\right] dx$$

$$= -\frac{\hbar^2}{a\sqrt{\pi}} \left[ \left(-\frac{1}{a^2} - k_0^2\right)\left(a\sqrt{\pi}\right) + \frac{1}{a^4}\,\frac{a^2}{2}\,a\sqrt{\pi} \right] =$$

$$= -\frac{\hbar^2}{a\sqrt{\pi}} \left[ -\frac{\sqrt{\pi}}{a} - k_0^2 a\sqrt{\pi} + \frac{\sqrt{\pi}}{2a} \right] = -\hbar^2\left[-\frac{1}{2a^2} - k_0^2\right] = \hbar^2 k_0^2 + \frac{\hbar^2}{2a^2}$$

$\Delta p = p - \langle p \rangle$ ; $(\Delta p)^2 = (p - \langle p \rangle)^2 = p^2 - 2p\langle p \rangle + \langle p \rangle^2$

$$\langle (\Delta p)^2 \rangle = \langle p^2 \rangle - 2\langle p \langle p \rangle \rangle + \langle p \rangle^2 = \langle p^2 \rangle - \langle p \rangle^2$$

$$\langle (\Delta p)^2 \rangle = \hbar^2 k_0^2 + \frac{\hbar^2}{2a^2} - (\hbar k_0)^2 = \frac{\hbar^2}{2a^2} \quad \text{OK}$$

v p-repre

$$\psi(k, 0) = \sqrt{\frac{a}{\sqrt{\pi}}} \exp\left[-a^2(k_0 - k)^2/2\right]$$

$p = \hbar k$ $\quad dp = \hbar dk$

$$\psi(p, 0) = \sqrt{\frac{a}{\sqrt{\pi}}} \exp\left[-a^2(p_0 - p)^2/(2\hbar^2)\right] \frac{1}{\hbar} \; ?$$

OK

$$\langle (p - \langle p \rangle)^2 \rangle = \hbar^2 \langle (k - \langle k \rangle)^2 \rangle = \hbar^2 \langle (k - k_0)^2 \rangle$$

$$= \hbar^2 \frac{a}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left(-a^2(k_0 - k)^2\right) (k - k_0)^2 \, dk = \hbar^2 \frac{a}{\sqrt{\pi}} \sqrt{\frac{\pi}{a^2}} \, \frac{1}{2a^2} = \frac{\hbar^2}{2a^2}$$

$x^2 \exp(-\alpha x^2)$

$$\langle (\Delta x)^2 \rangle \langle (\Delta p)^2 \rangle = \frac{a^2}{2} \cdot \frac{\hbar^2}{2a^2} = \frac{\hbar^2}{4} \quad \text{OK}$$

(relace neurčitosti splněna pro Gaussovku přesně)

$\psi = e^{ikr}$ ? $\Delta p = 0$, $\Delta x = \infty$ ?

ale $\psi = e^{ikr} e^{-\frac{x^2}{2a^2}}$ $\quad \langle (\Delta p)^2 \rangle = \frac{\hbar^2}{2a^2}$ $\quad \langle (\Delta x)^2 \rangle = \frac{a^2}{2}$ $\quad \langle (\Delta p)^2 \rangle \langle (\Delta x)^2 \rangle = \frac{\hbar^2}{4}$

→ pošleme a k nekonečnu a stále platí H. relace!

10) $\psi(t) = \sum_n c_n \psi_n \, e^{-iE_n t/\hbar} = \oint \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} c_k \psi_k e^{-iE_k t/\hbar}$ UKMT4.56

volné částice – vl. stavy jsou $\psi(k) = e^{ikx}$, s energií $E_n = \frac{\hbar^2 k^2}{2m}$

počáteční stav $\psi(x,0)$ přepíšeme pomocí $\psi(k)$

$$\psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} c(k)\, \psi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} c(k)\, e^{ikx}$$

$$c(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} \psi(x,0)$$

$$c(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} \frac{1}{\sqrt{a\sqrt{\pi}}} \exp\left[-\frac{x^2}{2a^2} + ik_0 x\right] dx$$ (viz příklad 7)

$$= \sqrt{\frac{a}{\sqrt{\pi}}} \exp\left[-a^2 (k_0 - k)^2 / 2\right]$$

$$\psi(x,t) = \sqrt{\frac{a}{2\pi}} \frac{1}{\pi^{1/4}} \int_{-\infty}^{\infty} \exp\left[-a^2 (k_0 - k)^2 / 2\right] e^{ikx} e^{-i\frac{\hbar k^2 t}{2m}} \, dk$$

$$= \sqrt{\frac{a}{2\pi}} \frac{1}{\pi^{1/4}} \int_{-\infty}^{\infty} \exp\left[-\frac{a^2 k^2}{2} + \cancel{2} a^2 k_0 k - \frac{a^2 k_0^2}{2} + ikx - i\frac{\hbar k^2 t}{2m}\right] = *$$

$$-\left[\frac{a^2 k^2}{2} + \frac{i\hbar k^2 t}{2m} - ikx - a^2 k_0 k + \frac{a^2 k_0^2}{2}\right]$$

$$= -\left[ k^2 \Big(\underbrace{\frac{a^2}{2} + \frac{i\hbar t}{2m}}_{A^2}\Big) - \underbrace{k\,(a^2 k_0 + ix)}_{2AB} + \frac{a^2 k_0^2}{2}\right]$$

$$= - \{$$

$$A^2 = \frac{a^2}{2} + \frac{i\hbar t}{2m} \qquad 2AB = a^2 k_0 + ix$$

$$A = \sqrt{\frac{a^2}{2} + \frac{i\hbar t}{2m}} \qquad B = \frac{a^2 k_0 + ix}{2\sqrt{\frac{a^2}{2} + \frac{i\hbar t}{2m}}} \qquad B^2 = \frac{(a^2 k_0 + ix)^2}{4\left(\frac{a^2}{2} + \frac{i\hbar t}{2m}\right)}$$

$$* = \sqrt{\frac{a}{2\pi}} \frac{1}{\pi^{1/4}} e^{-\frac{a^2 k_0^2}{2}} \int_{-\infty}^{\infty} \exp\left[-\left(k^2 \left(\frac{a^2}{2} + \frac{i\hbar t}{2m}\right) - k(a^2 k_0 + ix) + \frac{(a^2 k_0 + ix)^2}{4\left(\frac{a^2}{2} + \frac{i\hbar t}{2m}\right)}\right)\right] \cdot \exp\left[+\frac{(a^2 k_0 + ix)^2}{4\left(\frac{a^2}{2} + \frac{i\hbar t}{2m}\right)}\right] dk =$$

$$\int \exp[-(\alpha k)^2] = \frac{\sqrt{\pi}}{\alpha}$$

$$= \sqrt{\frac{a}{2\pi}} \frac{1}{\pi^{1/4}} e^{-\frac{a^2 k_0^2}{2}} \exp\left[\frac{(a^2 k_0 + ix)^2}{2\left(a^2 + \frac{i\hbar t}{m}\right)}\right] \int_{-\infty}^{\infty} \exp\left[-\left(\sqrt{\frac{a^2}{2} + \frac{i\hbar t}{2m}}\, k - \frac{a^2 k_0 + ix}{2\sqrt{\frac{a^2}{2} + \frac{i\hbar t}{2m}}}\right)^2\right] dk$$

$$= \sqrt{\frac{a}{2\pi}} \frac{1}{\pi^{1/4}} e^{-\frac{a^2 k_0^2}{2}} \exp\left[\frac{(a^2 k_0 + ix)^2}{2\left(a^2 + \frac{i\hbar t}{m}\right)}\right] \frac{\sqrt{\pi}}{\sqrt{\frac{a^2}{2} + \frac{i\hbar t}{2m}}} =$$

$$= \frac{1}{\pi^{1/4} \sqrt{a}} \frac{1}{\sqrt{1 + \frac{i\hbar t}{a^2 m}}} e^{-\frac{a^2 k_0^2}{2}} \cdot \exp\left[\frac{a^4 \left(k_0 + \frac{ix}{a^2}\right)^2}{2a^2 \left(1 + \frac{i\hbar t}{m a^2}\right)}\right]$$

$$\psi(x,t) = \frac{1}{\pi^{1/4}} \frac{1}{\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{a^2 m}}} e^{-\frac{a^2 k_0^2}{2}} \exp\left[\frac{a^2\left(k_0 + \frac{ix}{a^2}\right)^2}{2\left(1+\frac{i\hbar t}{ma^2}\right)}\right]$$

$$= \frac{1}{\pi^{1/4}\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{ma^2}}} e^{-\frac{a^2 k_0^2}{2}} \exp\left[\frac{a^2k_0^2 + 2ixk_0 - \frac{x^2}{a^2}}{2\left(1+\frac{i\hbar t}{ma^2}\right)}\right]$$

$$= \frac{1}{\pi^{1/4}\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{ma^2}}} e^{-\frac{a^2k_0^2\left(1+\frac{i\hbar t}{ma^2}\right)}{2\left(1+\frac{i\hbar t}{ma^2}\right)}} \exp\left[\frac{a^2k_0^2 + 2ixk_0 - \frac{x^2}{a^2}}{2\left(1+\frac{i\hbar t}{ma^2}\right)}\right]$$

$$= \frac{1}{\pi^{1/4}\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{ma^2}}} \exp\left[\frac{-\frac{x^2}{a^2} + 2ixk_0 - \frac{k_0^2 i\hbar t}{m}}{2\left(1+\frac{i\hbar t}{ma^2}\right)}\right]$$

$$= \frac{1}{\pi^{1/4}\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{ma^2}}} \exp\left[\frac{\left(-\frac{x^2}{2a^2} + ik_0x - \frac{k_0^2 i\hbar t}{2m}\right)\left(1-\frac{i\hbar t}{ma^2}\right)}{\left(1+\frac{i\hbar t}{ma^2}\right)\left(1-\frac{i\hbar t}{ma^2}\right)}\right]$$

$$= \frac{1}{\pi^{1/4}\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{ma^2}}} \exp\left[\frac{-\frac{x^2}{2a^2} + ik_0x - \frac{k_0^2 i\hbar t}{2m} + \frac{i\hbar t x^2}{2ma^4} + \frac{k_0\hbar t x}{ma^2} - \frac{k_0^2\hbar^2t^2}{2m^2a^2}}{1+\frac{\hbar^2t^2}{m^2a^4}}\right]$$

$$= \frac{1}{\pi^{1/4}\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{ma^2}}} \exp\left[\frac{-\left(\frac{x^2}{2a^2} - \frac{k_0\hbar t x}{ma^2} + \frac{k_0^2\hbar^2t^2}{2m^2a^2}\right) + i\left(\frac{\hbar t x^2}{2ma^4} + k_0x + \frac{k_0^2\hbar t}{2m}\right)}{1+\frac{\hbar^2t^2}{m^2a^4}}\right]$$

← spiš toto

$$= \frac{1}{\pi^{1/4}\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{ma^2}}} \exp\left[\frac{-\frac{1}{2a^2}\left(x - \frac{k_0\hbar t}{m}\right)^2 + ik_0x + i\frac{\hbar t}{2m}\left(\frac{x^2}{a^4} + k_0^2\right)}{1+\frac{\hbar^2t^2}{m^2a^4}}\right]$$

$$\frac{x^2}{a^4} + k_0^2 + 2\frac{k_0x}{a^2} - \frac{2k_0x}{a^2} = \left(\frac{x}{a^2} + k_0\right)^2 - \frac{2k_0x}{a^2} \qquad i\frac{\hbar t}{2m}\frac{2k_0x}{a^2} = \frac{i\hbar t k_0 x}{ma^2}$$

$$= \frac{1}{\pi^{1/4}\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{ma^2}}} \exp\left[\frac{-\frac{1}{2a^2}\left(x - \frac{k_0\hbar t}{m}\right)^2 + ik_0x\left(1 - \frac{\hbar t}{ma^2}\right) + \frac{i\hbar t}{2m}\left(\frac{x}{a^2} + k_0\right)^2}{1+\frac{\hbar^2t^2}{m^2a^4}}\right]$$

rovinná vlna, změna fáze, gaussián

posun středu gaussiánu $s = \frac{p}{m}t = v\,t$

11) $$\psi = \frac{1}{\pi^{1/4}\sqrt{a}} \frac{1}{\sqrt{1+\frac{i\hbar t}{ma^2}}} \exp\left[\frac{-\frac{1}{2a^2}\left(x - \frac{k_0\hbar t}{m}\right)^2 + ik_0x + \frac{i\hbar t}{2m}\left(\frac{x^2}{a^4}+k_0^2\right)}{1+\frac{\hbar^2t^2}{m^2a^4}}\right]$$

OKAY TY.6b

$$\rho = \psi^*\psi = \frac{1}{\sqrt{\pi}\,a}\frac{1}{\sqrt{\left(1+\frac{i\hbar t}{ma^2}\right)\left(1-\frac{i\hbar t}{ma^2}\right)}} \exp\left[-\frac{\frac{(x-k_0\hbar t/m)^2}{a^2}}{1+\frac{\hbar^2t^2}{m^2a^4}}\right]$$

$$= \frac{1}{\sqrt{\pi}\,a}\frac{1}{\sqrt{1+\frac{\hbar^2t^2}{m^2a^4}}}\exp\left[-\frac{(x-k_0\hbar t/m)^2}{a^2+\frac{\hbar^2t^2}{m^2a^2}}\right]$$

$\sim \frac{1}{\sqrt{\pi}}\alpha\, \exp(-\alpha x^2)$

$\sim \alpha\, \exp(-\alpha(x-x_0)^2)$

→ maximum se posunuje s $\frac{k_0\hbar t}{m}$

→ šířka roste $a^2\left(1+\frac{\hbar^2t^2}{m^2a^4}\right)$ $\sim a(t) = a_0\sqrt{1+\frac{\hbar^2t^2}{m^2a^4}}$

$$j = \frac{\hbar}{2mi}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right) =$$

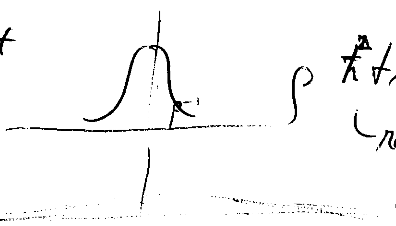
+ 2. člen

$$= \frac{\hbar}{2mi}\psi^*\psi\left[\left(\frac{ik_0 + \frac{i\hbar t x}{ma^4}}{1+\frac{\hbar^2t^2}{m^2a^4}}\right)\cdot 2\right] = \frac{\hbar k_0}{m}\psi^*\psi\left[\frac{1+\frac{\hbar t x}{ma^4k_0}}{1+\frac{\hbar^2t^2}{m^2a^4}}\right]$$

$k_0 = 0$ $$\to \frac{\hbar}{m}\psi^*\psi\frac{\cancel{k_0} + \frac{\hbar t x}{ma^4}}{1+\frac{\hbar^2t^2}{m^2a^4}} = \rho\frac{\frac{\hbar^2 t x}{m^2a^4}}{\frac{m^2a^4+\hbar^2t^2}{m^2a^4}} = \rho\frac{\hbar^2 t x}{m^2a^4+\hbar^2t^2}$$

hustota v čase $t$!

$t = 0^+$



$\rho\, \hbar t x$

rozplizování v prostoru úměrné vzdálenosti od $0$

$\rho$ pro $x=0$ : $$\frac{1}{\sqrt{\pi}\,a}\frac{1}{\sqrt{1+\frac{\hbar^2t^2}{m^2a^4}}}\exp\left[-\frac{(\overset{x=0}{0}-\overset{k_0=0}{0})^2}{(\quad)}\right]$$ ← jen pokles v čase

↳ z FT $$\psi(\overset{x=}{0},t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} N e^{-\frac{a^2k^2}{2} - \frac{i\hbar k^2 t}{2m}}dk$$

$1/x$ - liše

- vlny $e^{ikx}$ mění fázi s $e^{-iE_nt/\hbar}$ faktorem
- vln je nekonečný počet a $E_n$ je spojité a nekonečně malé →
  → nikdy se nesfázují dohromady zpět!!! (narozdíl od třeba 2-stavového systému)