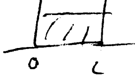
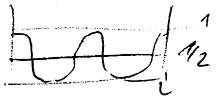
 periodic $\psi_0 = \frac{1}{\sqrt{L}}$ & $\sin\left(\frac{2\pi n x}{L}\right)$



$$\psi_n = \cancel{\frac{1}{\sqrt{L}}} N \cos\left(\frac{2\pi n x}{L}\right)$$

$$\langle \psi_1 | \psi_1 \rangle = N^2 \int_0^L \cos^2\left(\frac{2\pi x}{L}\right) dx = N^2 \int_0^L \frac{1}{4}\left[2 + e^{\frac{4\pi i x}{L}} + e^{-\frac{4\pi i x}{L}}\right] dx = *$$

$$\cos\left(\frac{2\pi x}{L}\right) = \frac{1}{2}\left[e^{\frac{2\pi i x}{L}} + e^{-\frac{2\pi i x}{L}}\right]$$

$$\cos^2\left(\frac{2\pi x}{L}\right) = \frac{1}{4}\left[2 + e^{\frac{4\pi i x}{L}} + e^{-\frac{4\pi i x}{L}}\right]$$

$$* = \frac{N^2}{4}\int_0^L \left(2 + e^{\frac{4\pi i x}{L}} + e^{-\frac{4\pi i x}{L}}\right) dx = \frac{N^2}{4}\left[2L + 0 + 0\right] = 1$$

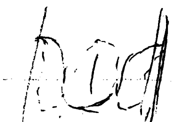
$$N^2 = \frac{2}{L}$$

$$N = \sqrt{\frac{2}{L}} \quad \text{OK}$$

$$V' = A\,\delta(x - L/2)$$

$$\langle m | V' | n' \rangle = \frac{2}{L}\int_0^L \cos\left(\frac{2\pi m x}{L}\right) \delta\left(x - \frac{L}{2}\right) \cos\left(\frac{2\pi n x}{L}\right) =$$

——— e^{inkx} nicer...



$$\psi_n = N e^{i\frac{2\pi}{L} n x} \qquad E = \frac{\hbar^2}{2m}\frac{4\pi^2}{L^2} n^2 = \frac{2\hbar^2\pi^2 n^2}{m L^2}$$

$$\langle n | n \rangle = N^2 \int_0^L e^{-\frac{i2\pi}{L} n x} e^{\frac{2\pi i}{L} n x} dx = N^2 \int_0^L dx = N^2 L = 1$$

$$N = \frac{1}{\sqrt{L}}$$

$$V' = A\,\delta(x - L/2)$$

$$\langle m | V' | n \rangle = \frac{A}{L}\int_0^L e^{-\frac{2\pi i}{L} m x}\, \delta\left(x - \frac{L}{2}\right) e^{\frac{2\pi i}{L} n x} dx =$$

$$= \frac{A}{L}\int_0^L \delta\left(x - \frac{L}{2}\right) e^{\frac{2\pi i}{L} x (n - m)} dx =$$

$$= \frac{A}{L} e^{\frac{2\pi i}{L}\frac{L}{2}(n-m)} = \frac{A}{L} e^{i\pi(n-m)} \begin{cases} = 1 & (n-m) \text{ even} \\ = -1 & (n-m) \text{ odd} \end{cases}$$

$$\Delta E_0^{(1)} = \langle 0 | V' | 0 \rangle = \frac{A}{L}$$

$$\Delta E_0^{(2)} = \sum_{j'} \frac{|\langle j' | V' | 0 \rangle|^2}{\varepsilon_0 - \varepsilon_j} = \sum_{j'} \frac{\frac{A^2}{L^2}}{0 - \frac{2\hbar^2\pi^2 j^2}{m L^2}} = -\frac{A^2 m}{2\hbar^2\pi^2}\sum_{\substack{j \neq 0}} \frac{1}{j^2} =$$

$$= -\frac{A^2 m}{2\hbar^2\pi^2} \cdot 2 \cdot \sum_{j=1}^{\infty}\frac{1}{j^2} = -\frac{A^2 m}{\hbar^2\pi^2}\cdot\frac{\pi^2}{6} = -\frac{A^2 m}{\hbar^2 6}$$

$\Delta E_n^{(1)}$ - twice degenerate $\frac{A}{L}\begin{pmatrix} 1 & +1 \\ +1 & 1 \end{pmatrix} \to \frac{A}{L} \pm \frac{A}{L}$ $\Rightarrow 0$ $\quad \frac{1}{\sqrt{2}}(|n\rangle - |-n\rangle) \to \sin$

$\Rightarrow \frac{2A}{L} \quad \frac{1}{\sqrt{2}}(|n\rangle + |-n\rangle) \to \cos$

$$|\psi_i^{(1)}\rangle = |\psi_i^{(0)}\rangle + \sum_{j\neq i}^{\infty} \frac{|j\rangle\langle j|V'|i\rangle}{\varepsilon_i - \varepsilon_j}$$

$$= |i\rangle + \sum_{j\neq i}^{\infty} \frac{|j\rangle \frac{A}{L} e^{i\pi(j-i)}}{\varepsilon_i - \varepsilon_j}$$

$i=0$

$$= \frac{1}{\sqrt{L}} + \sum_{j\neq i}^{\infty} \frac{\frac{1}{\sqrt{L}} e^{\frac{2\pi i j x}{L}} \frac{A}{L} e^{i\pi(j-\overset{i=0}{i})}}{0 - \frac{2\hbar^2\pi^2 j^2}{m L^2}}$$

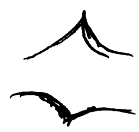
$$= \frac{1}{\sqrt{L}} + \frac{mL^2}{2\hbar^2\pi^2} \cdot \frac{1}{\sqrt{L}} \cdot \frac{A}{L} \sum_{j\neq i}^{\infty} \frac{e^{\frac{2\pi i j x}{L}} e^{i\pi(j-\overset{i=0}{i})}}{j^2}$$

$$= \frac{1}{\sqrt{L}} - \frac{mAL}{2\hbar^2\pi^2\sqrt{L}} \sum_{\substack{j\neq i \\ -\infty}}^{\infty} \frac{e^{\frac{2\pi i j x}{L}} e^{i\pi(j-\overset{i=0}{i})}}{j^2}$$

$$= \frac{1}{\sqrt{L}} - \frac{mAL}{2\hbar^2\pi^2\sqrt{L}} \sum_{j=1}^{\infty} \frac{e^{i\pi(j-\overset{0}{i})}\left[e^{\frac{2\pi i j x}{L}} + e^{-\frac{2\pi i j x}{L}}\right]}{j^2} =$$

$$= \frac{1}{\sqrt{L}} - \frac{mAL}{2\hbar^2\pi^2\sqrt{L}} \sum_{j=1}^{\infty} \frac{e^{i\pi j} \cos\left(\frac{2\pi j x}{L}\right)}{j^2}$$

$$= \frac{1}{\sqrt{L}} - \frac{mAL}{2\hbar^2\pi^2\sqrt{L}} \sum_{j=1}^{\infty} \frac{(-1)^j \cos\left(\frac{2\pi j x}{L}\right)}{j^2} \rightarrow$$



cusp
really infinity required...

for $x = \frac{L}{2}$: $\frac{1}{\sqrt{L}} - \frac{mAL}{2\hbar^2\pi^2\sqrt{L}} \sum_{j=1}^{\infty} \frac{(-1)^j \cos(\pi j)}{j^2}$ $(-1)^j$

$$= \frac{1}{\sqrt{L}} - \frac{mAL}{2\hbar^2\pi^2\sqrt{L}} \sum_{j=1}^{\infty} \frac{1}{j^2}$$

second derivative - infinite

LHO + $\frac{1}{2}m\omega^2x^2$ pert. $\frac{1}{2}m\omega^2 = m\omega^2$ Pert-A-5

$$H_0 = -\frac{\hbar^2 d^2}{2m\,dx^2} + \frac{1}{2}m\omega^2x^2 \qquad \omega'^2 = 2\omega^2 \qquad \omega' = \sqrt{2}\omega \rightarrow E = \frac{1}{2}\hbar\omega\cdot\sqrt{2}$$

$$V' = \frac{1}{2}m\omega^2x^2$$

crucial, HA!

1st order $\langle\emptyset|V'|\emptyset\rangle = \langle\cancel{\emptyset|V'|\emptyset}\ \langle\emptyset|\frac{1}{2}m\omega^2x^2|\emptyset\rangle = \boxed{\frac{1}{4}\hbar\omega}$

$$= \frac{1}{2}m\omega^2\langle\emptyset|x^2|\emptyset\rangle = *$$

$$x = \frac{\alpha}{\sqrt{2}}(a+a^+) \qquad \alpha = \sqrt{\frac{\hbar}{m\omega}} \qquad aa^+ = 1 + a^+a$$

$$x^2 = \frac{\alpha^2}{2}(a+a^+)(a+a^+) = \frac{\alpha^2}{2}(a^2+a^{+2}+aa^++a^+a) =$$
$$= \frac{\alpha^2}{2}(a^2+a^{+2}+2a^+a+1)$$

$$* = \frac{1}{2}m\omega^2\cdot\frac{\hbar}{2\cdot m\omega}\langle\emptyset|a^2+a^{+2}+\cancel{2a^+a}+1|\emptyset\rangle =$$

$$= \frac{1}{4}\hbar\omega\cdot 1 \quad \leftarrow \text{OK} \qquad \frac{1}{2}\hbar\omega\left(1+\frac{1}{2}\right) \stackrel{!}{=} \frac{1}{2}\hbar\omega\cdot 1{,}5 \text{ vs } \cdot\sqrt{2}$$

getting these

only $j=2$!

$$2^{nd}\text{ order: } \Delta E_\emptyset^{(2)} = \sum_{j\neq 0}\frac{|\langle j|V'|\emptyset\rangle|^2}{\varepsilon_0-\varepsilon_j} = \frac{|\langle 2|V'|\emptyset\rangle|^2}{\varepsilon_0-\varepsilon_2}$$

$$\langle 2|V'|\emptyset\rangle = \frac{1}{2}m\omega^2\cdot\frac{\alpha^2}{2}\langle 2|a^2+a^{+2}+2a^+a+1|\emptyset\rangle =$$
$$= \frac{1}{4}m\omega^2\frac{\hbar}{m\omega}\langle 2|a^{+2}|\emptyset\rangle = \frac{1}{4}\hbar\omega\langle 2|a^+\sqrt{1}|1\rangle =$$
$$= \frac{1}{4}\hbar\omega\sqrt{2}\cdot 1$$

$$\Delta E = \frac{|\frac{\sqrt{2}}{4}\hbar\omega|^2}{\varepsilon_0-\varepsilon_2} = \frac{\frac{1}{8}\hbar^2\omega^2}{\frac{1}{2}\hbar\omega-\frac{5}{2}\hbar\omega} = -\frac{\hbar\omega}{16}$$

through 2^{nd} order : $\frac{1}{2}\hbar\omega + \frac{1}{4}\hbar\omega - \frac{\hbar\omega}{16} = \frac{1}{2}\hbar\omega\left(1+\frac{1}{2}-\frac{1}{8}\right) =$

$\frac{1}{2}\hbar\omega\cdot 1{,}375$ vs $1{,}41 \approx \sqrt{2}$

Pert-A-5b $\left(\frac{1}{4}\hbar\omega\sqrt{2}\right)^2$

third order:

$$\sum_{k\neq n}\sum_{m\neq n}\frac{\langle n|V'|m\rangle\langle m|V'|k\rangle\langle k|V'|n\rangle}{\Delta E_{mn}\,\Delta E_{kn}} - \langle n|V'|n\rangle\sum_{m\neq n}\frac{|\langle n|V'|m\rangle|^2}{\Delta E_{mn}^2}$$

$(2\hbar\omega)^2$

$\frac{1}{4}\hbar\omega$

$$\frac{\langle 0|V'|2\rangle\langle 2|V'|2\rangle\langle 2|V'|0\rangle}{\Delta E_{20}\,\Delta E_{20}}$$

$$= \frac{\frac{1}{4}\hbar\omega\sqrt{2}\cdot\frac{5}{4}\hbar\omega\cdot\frac{1}{4}\hbar\omega\sqrt{2}}{(2\hbar\omega)^2} =$$

$$= -\frac{1}{4}\hbar\omega\cdot\frac{\frac{1}{16}\hbar^2\omega^2\cdot 2}{4\hbar^2\omega^2} = -\frac{\hbar\omega}{128}$$

$$= \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{2}{4}\cdot 5\;\hbar\omega =$$

$$= \frac{5}{128}\cdot\hbar\omega$$

$$\frac{5}{128}\cdot\hbar\omega - \frac{\hbar\omega}{128} = \cancel{\frac{19}{128}\hbar\omega} = \cancel{0.1484}$$

$$= \cancel{\frac{1}{2}\hbar\omega\frac{19}{64}} = \frac{4}{128}\cdot\hbar\omega = \frac{\hbar\omega}{32}$$

through 3rd order

$$\frac{1}{2}\hbar\omega\left(1+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}\ldots\right) = \cancel{\frac{1}{2}\hbar\omega\left(1{,}375+\frac{19}{64}\right)} =$$

$$= \cancel{\frac{1}{2}\hbar\omega\cdot 1{,}6719} \leftarrow \text{a bit more off}$$

$$= \frac{1}{2}\hbar\omega\,(1{,}375+0{,}0625) = \frac{1}{2}\hbar\omega\cdot 1{,}4375$$

order	E	ΔE
1	$\frac{1}{2}\hbar\omega\cdot 1{,}5$	0,0858
2	$\frac{1}{2}\hbar\omega\cdot 1{,}375$	-0,0392
3	$\frac{1}{2}\hbar\omega\cdot 1{,}4375$	0,0233

wiki

$\frac{5}{16\cdot 8} = \frac{5}{128}$

agrees

$$\sqrt{2} = \sum_{k=0}^{\infty}(-1)^{k+1}\frac{(2k-3)!!}{(2k)!!} = 1+\frac{1}{2}-\frac{1}{2\cdot 4}+\frac{1\cdot 3}{2\cdot 4\cdot 6}-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}+\ldots$$